

# Between writing and picturing: The square of opposition in formal logic and formal axiology of alethic and deontic modalities

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From some point of view, the famous "Square of Opposition" in the formal logic has nothing to do with squares and other geometrical objects which can be pictured, shown and seen. All the rational formal logic contents of the "Square of Opposition" could be written, read, said and understood without any picturing and showing. The well-known picture is really useful only in mnemonic relation: the scheme supports the mimesis. Assuming this abstract-language-form viewpoint in the present paper, and using the hypothetic-deductive method, below I develop the mentioned assumption (hypothesis) with respect to one particular case, namely, in relation to alethic and deontic modality interpretations of the "Square". According to the hypothesis under examination, all the formal rules making up the rational contents of the "Square" are to be adequately formulated and expressed in a written language without picturing.

Which language is implied above: the artificial or natural one? According to Wittgenstein both language types entail problems. I agree with Wittgenstein and hence before attacking the specific topic of this paper it is worth making some remarks concerning linguistic philosophy. The paper submits quite a new idea of mathematical philosophy of formal semantics of natural languages. I think that it is not adequate completely to reduce the formal semantics of natural languages to their formal logic one dealing with descriptive-indicative meanings. In complement to the formal logic semantics there is a formal axiological one dealing with evaluative meanings – moral-legal evaluation-functions (Lobovikov 2007; 2009a; 2009b). However, mathematical philosophy of formal logic semantics is our starting point. Moving towards a formal axiological semantics one has to depart from the formal logic one. According to the history of philosophy, originally just formal logic generated the precedent of mathematical philosophy. This historical fact was necessary because at first only logic was recognized as formal one. All the rest philosophical disciplines were considered as material ones on principle. However, today the situation is significantly changed. Contemporary information technologies require a substantially new paradigm of philosophizing. A kind of digital philosophy is required. In this respect it is worth inventing and elaborating a digital simulation of metaphysics and ethics (a mathematical representation of morals). Nowadays, in spite of the pessimistic beliefs and expectations, the mathematical philosophy of morals (symbolic ethics) is naturally emerging from the contemporary symbolic logic – the mathematical philosophy of thought. This is so because, according to prominent logicians and philosophers Charles Sanders Peirce, Jan Łukasiewicz and Frank Ramsay, "logic is nothing but morals of thinking". Thinking is a particular case (kind) of human acting. Hence algebraic system of thoughts is a subsystem of an algebraic system of acts. The "true" is a particular case of the "good". The "false" is a particular case of the "bad". Starting from this heuristically important basis in present paper I construct and investigate a discrete mathematical representation of the rigorist ("black-and-white") system of

formal ethics. Thus I prove that, in principle, there is a possibility to go from machine simulation of formal logic of human thinking to machine simulation of formal ethics of human conduct (Lobovikov 2009a).

The possibility of machine simulation of human moral-legal conduct is demonstrated by the basic mathematical ethics – two-valued algebra of good and evil. This algebra is based upon the set of acts. By definition, acts are such and only such operations, which are either good, or bad ones. Algebraic operations defined on the set of acts are moral-legal evaluation functions. Variables of these functions take their values from the set  $\{g, b\}$ . The functions take their values from the same set. Symbols  $g$  and  $b$  stand for moral-legal values of acts "good" and "bad" respectively. The symbols:  $x$  and  $y$  stand for moral-legal forms of acts. Elementary moral-legal act forms – independent variables. Compound moral-legal act forms – moral-legal evaluation functions of these variables. In the two-valued algebra there are 16 mathematically different binary operations defined by tables 1 and 2 (see Appendix).

The symbol  $Kxy$  stands for the moral-legal operation "combining (uniting)  $x$  and  $y$  (in the conduct as a whole)".  $Sxy$  – moral-legal operation "separating  $x$  and  $y$ ".  $Axy$  – "non-excluding choice of the best among the acts made up by  $x$  and  $y$ ".  $Wxy$  – "abstaining from both  $x$  and  $y$ ".  $Uxy$  – "excluding choice of the best between the acts  $x$  and  $y$ ".  $Txy$  – "identifying  $x$  and  $y$ ".  $Hxy$  – "offensive, attack, assault of  $y$  on (against)  $x$ ".  $Dxy$  – "defending  $x$  from (against)  $y$ ". Symbol  $Cxy$  stands for the moral-legal operation "doing  $y$  in response to  $x$ ".  $Vxy$  – "counter-offensive (counter-attack) of  $x$  on  $y$ ".  $Lxy$  – "independence (freedom) of  $x$  from  $y$ ".  $Qxy$  – "independence (freedom) of  $y$  from  $x$ ".  $Fxy$  – "independence of destruction (termination) of  $x$  from  $y$ ".  $Ixy$  – "independence of destruction (termination) of  $y$  from  $x$ ".  $Gxy$  – identically (constantly) good moral-legal form. (This kind of forms represents immutable universal laws of morals, which are common for all times and peoples).  $Zxy$  – identically (constantly) bad moral-legal form. (It represents moral-legal evaluation forms of violating immutable universal laws of morals).

Below some unary moral-legal operations of algebra of ethics are considered. Symbol  $Ex$  stands for "consolidation, unity of  $x$ ".  $Rx$  – "division (split) of  $x$ ".  $Bx$  – "abstaining from  $x$ ".  $Yx$  – "independence (freedom) of  $x$ ".  $Jx$  – "destruction (termination) of  $x$ ".  $Xx$  – "violence against  $x$ ".  $Px$  – "punishment (retribution) for  $x$ ".  $Mx$  – "encouragement (award) for  $x$ ".  $Nx$  – "non-existence (non-being) of  $x$ ".  $Ox$  – "blame for  $x$ ".  $A^*x$  – "approval of  $x$ ".  $I^*x$  – "(moral-legal) indifference to  $x$ ", i.e., "unprincipled attitude to  $x$ ".  $P^*x$  – "principled attitude to  $x$ ", i.e., "adherence to (moral-legal) principles in relation to  $x$ ". The evaluation-functional sense of these operations is defined by table 3 (see Appendix).

In algebra of formal ethics the equivalence relation is defined as follows. Moral-legal act forms  $x$  and  $y$  are formally-ethically equivalent (this is represented by

the symbol " $x=+=y$ ", if and only if they acquire identical moral-legal values (from the set  $\{g, b\}$ ) under any possible combination of moral-legal values of the variables. By means of the above-given definitions one can obtain interesting equations: 1)  $Ex=+=Kxx$ . 2)  $Rx=+=Jx$ . 3)  $Ix=+=BAxBx$ . 4)  $Rx=+=Sxx$ . 5)  $Px=+=Xx$ . 6)  $Ix=+=KOxMx$ . 7)  $Bx=+=Wxx$ . 8)  $Px=+=Cxb$ . 9)  $Ix=+=KAxPx$ . 10)  $Yx=+=Lxy$ . 11)  $Mx=+=BPx$ . 12)  $Nxx=+=XXx$ . 13)  $Jx=+=Fxy$ . 14)  $Mx=+=HBxg$ . 15)  $Px=+=AxBx$ . This list is open.

Taking the above considerations seriously, it is easy to see that there is a fundamental interconnection between algebra of logic and algebra of ethics. From the two-valued ethics viewpoint, there is a metaphysical unity of the "true" and the "good" acts: true is good (but, generally speaking, the converse statement is false). And also the similar (analogous) type of unity exists between the "false" and the "bad": lie is evil (but, generally speaking, the converse statement is false). Consequently, from the two-valued ethics viewpoint, the truth-tables of Boolean algebra of logic are particular cases of corresponding moral-legal evaluation tables of two-valued algebra of the rigorist formal ethics. This is because thought activity studied by logic is a particular case of activity (in general) studied by ethics (in its own specific relation).

In the present paper I discuss a specific theme of formal ethics as generalization of formal logic, namely, "a square of opposition of moral-legal evaluation functions in formal ethics" as the formal-axiological generalization of the square of opposition in formal logic. Moreover in the given paper I have restricted my investigation to alethic and deontic interpretations of the system of formal rules called "Square of Opposition". Let us start investigating such a severely reduced paper domain by dealing with its part – the alethic-modality interpretation. For doing this, in the given paper departing from the modalities of thought to modalities of activity in general, I submit a novel modal interpretation of the "square of opposition" by means of treating the alethic modalities "necessity", "possibility", "impossibility", "being avoidable" and "contingency" as *moral-legal evaluation functions determined by two variables* (Lobovikov 2007). As in the two-valued algebra of formal axiology there are 16 mathematically different binary operations (defined by moral-legal evaluation-tables in (Lobovikov 2009a; 2009b)), all the *alethic-modality-making* operations considered as *binary moral-legal evaluation-functions* are among the 16. Let us introduce and define the *alethic-modality-making* operations as moral-legal evaluation-functions by means of the below glossary.

*The glossary for the following table A:* The symbol  $Lxy$  stands for the moral-legal evaluation function "making  $y$  necessary for  $x$ ". The symbol  $Mxy$  stands for "making  $y$  possible for  $x$ ".  $Sxy$  stands for "making  $y$  impossible for  $x$ ".  $Uxy$  – "making  $y$  avoidable (not-necessary) for  $x$ ".  $Axy$  – "making  $y$  accidental for  $x$ ".  $Dxy$  – "y's being *alethically determined (alethically not-neutral)* for  $x$ ".  $Vxy$  stands for "y's violence to (against)  $x$ ". The above-mentioned moral-legal evaluation-functions are precisely defined by table A (see Appendix).

Using the above-given definitions, it is easy to demonstrate the following.

(16) The *subordination* between "necessary" and "possible" exists: the moral-legal function  $CLxyMxy$  is *formally-axiologically (constantly) good* one, but  $CMxyLxy$  is not.

(17) The *subordination* between "impossible" and "avoidable" exists: the moral-legal function  $CSxyUxy$  is

*formally-axiologically (constantly) good* one, but  $CUxySxy$  is not.

(18) The *opposition* (contradictoriness) between "necessary" and "avoidable" exists: the moral-legal functions  $NTLxyUxy$ ,  $ULxyUxy$ ,  $ALxyUxy$  are *formally-axiologically (constantly) good* ones.

(19) The *opposition* (contradictoriness) between "impossible" and "possible" exists: the moral-legal functions  $NTSxyMxy$ ,  $USxyMxy$ ,  $ASxyMxy$  are *formally-axiologically (constantly) good* ones.

20) The *contrariness* between "necessary" and "impossible" exists: the moral-legal function  $NKLxySxy$  is *formally-axiologically (constantly) good* one.

21) The *sub-contrariness* between "possible" and "avoidable" exists: the moral-legal functions  $NKNMxyNUxy$ ,  $AMxyUxy$  are *formally-axiologically (constantly) good* ones.

The above statements (16-21) make up "the alethic interpretation" of the system of formal rules called "Square of Opposition". But it is very interesting that the statements (16-21) are *formal-axiological* ones. This is in accordance with the main idea of the present article.

Taking into an account that in the given paper I have limited (reduced) my investigation to alethic and deontic interpretations of the so-called Square of Opposition, let us begin investigating the second part of this severely reduced domain – the *deontic* interpretation of the "Square". As in the given paper I depart from the modalities of thought to modalities of activity in general, below I submit a novel modal interpretation of the "square of opposition" by means of treating the deontic modalities "obligatory", "permitted", "forbidden", "facultative" and "deontically indifferent (normatively not-regulated)" as *moral-legal evaluation functions determined by two variables* (Lobovikov 2007). As in the two-valued algebra of formal axiology there are 16 mathematically different binary operations [defined by moral-legal evaluation-tables in (Lobovikov 2009a; 2009b)], all the *deontic-modality-making* operations considered as *binary moral-legal evaluation-functions* are among the 16. Let us introduce and define the *deontic-modality-making* operations as moral-legal evaluation-functions by means of the below glossary and evaluation-table.

*The glossary for table B (see Appendix):* Below the symbol  $Oxy$  stands for the moral-legal evaluation function "making  $y$  obligatory for  $x$ ". The symbol  $Pxy$  stands for "making  $y$  permitted for  $x$ ".  $Fxy$  stands for "making  $y$  forbidden for  $x$ ".  $Yxy$  – "making  $y$  facultative (not-obligatory) for  $x$ ".  $Ixy$  – "making  $y$  deontically neutral (normatively indifferent) for  $x$ ".  $Rxy$  – "y's being *normatively regulated (deontically not-neutral), principled* for  $x$ ".  $Vxy$  stands for "y's violence to (against)  $x$ ". The above-mentioned moral-legal evaluation-functions are precisely defined by the following table 2.

By means of the above definitions, it is possible to substantiate the following statements.

(22) The *subordination* between "obligatory" and "permitted" exists: the moral-legal function  $COxyPxy$  is *formally-axiologically (constantly) good* one, but  $CPxyOxy$  is not.

(23) The *subordination* between "forbidden" and "facultative" exists: the moral-legal function  $CFxyYxy$  is *formally-axiologically (constantly) good* one, but  $CYxyFxy$  is not.

(24) The *opposition* (contradictoriness) between “obligatory” and “facultative” exists: the moral-legal functions  $NTO^{\circ}xyY^{\circ}xy$ ,  $UO^{\circ}xyY^{\circ}xy$ ,  $AO^{\circ}xyY^{\circ}xy$  are *formally-axiologically (constantly) good ones*.

(25) The *opposition* (contradictoriness) between “forbidden” and “permitted” exists: the moral-legal functions  $NTF^{\circ}xyP^{\circ}xy$ ,  $UF^{\circ}xyP^{\circ}xy$ ,  $AF^{\circ}xyP^{\circ}xy$  are *formally-axiologically (constantly) good ones*.

26) The *contrariness* between “obligatory” and “forbidden” exists: the moral-legal function  $NKO^{\circ}xyF^{\circ}xy$  is *formally-axiologically (constantly) good one*.

27) The *sub-contrariness* between “permitted” and “facultative” exists: the moral-legal functions  $NKNP^{\circ}xyNY^{\circ}xy$ ,

$AP^{\circ}xyY^{\circ}xy$  are *formally-axiologically (constantly) good ones*.

The above statements (22-27) make up “the deontic interpretation” of the system of formal rules called “Square of Opposition”. But it is very interesting that the statements (22-27) are *formal-axiological ones*. Thus the system of formal-axiological statements about the deontic modalities of activity is necessary and sufficient for strict formulation of the system of formal rules called “Square of Opposition (of the deontic modalities)”. Hence drawing the picture of square is excessive. However the picture of square is very useful for quick and effective memorizing and remembering the formal rules. The square is psychologically significant for mimesis and hence pedagogically important for knowledge-organizing.

**Appendix**

Table 1. Binary operations

x	y	Kxy	Sxy	Axy	Wxy	Uxy	Txy	Hxy	Dxy
g	g	g	b	g	b	b	g	b	g
g	b	b	g	g	b	g	b	b	g
b	g	b	g	g	b	g	b	g	b
b	b	b	g	b	g	b	g	b	g

Table 2. Binary operations

x	y	Cxy	Vxy	Lxy	Qxy	Fxy	Ixy	Gxy	Zxy
g	g	g	b	g	g	b	b	g	b
g	b	b	g	g	b	b	g	g	b
b	g	g	b	b	g	g	b	g	b
b	b	g	b	b	b	g	g	g	b

Table 3. Unary operations

x	Ex	Rx	Bx	Yx	Jx	Xx	Px	Mx	Nx	Ox	A <sup>1</sup> x	I <sup>1</sup> x	P <sup>1</sup> x
g	g	b	b	g	b	b	b	g	b	b	g	b	g
b	b	g	g	b	g	g	g	b	g	g	b	b	g

Table A: *Alethic-modality-making operations* as moral-legal evaluation-functions determined by two variables

x	y	L <sup>A</sup> xy	M <sup>A</sup> xy	S <sup>A</sup> xy	U <sup>A</sup> xy	A <sup>A</sup> xy	D <sup>A</sup> xy	V <sup>A</sup> xy
g	g	b	g	b	g	g	b	b
g	b	b	g	b	g	g	b	b
b	g	g	g	b	b	b	g	g
b	b	b	b	g	g	b	g	b

Table B: *Daeontic-modality-making operations* as moral-legal evaluation-functions determined by two variables

x	y	O <sup>D</sup> xy	P <sup>D</sup> xy	F <sup>D</sup> xy	Y <sup>D</sup> xy	I <sup>D</sup> xy	R <sup>D</sup> xy	V <sup>D</sup> xy
g	g	b	g	b	g	g	b	b
g	b	b	g	b	g	g	b	b
b	g	g	g	b	b	b	g	g
b	b	b	b	g	g	b	g	b

**Literature**

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