A Formal Analysis of Selected Proofs by Aquinas for the Uniqueness of God

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1. Introduction

In this paper I analyze selected proofs for the uniqueness of God, reconstructed from the writings of St. Thomas Aquinas. Contemporary philosophers are interested mainly in his proofs for the existence of God, but, interestingly, Aquinas himself attached no lesser importance to the proofs for the uniqueness of God: for example, in Summa Contra Gentiles he gives us seventeen arguments for this claim.

We shall not consider problems such as the Holy Trinity, though this topic is not without interest for philosophical explorations (which could amount in this case mainly to analyzing purported compatibility of the Trinitarian thesis with monotheism). We shall reconstruct the arguments using proofs for the uniqueness of God stated directly by Aquinas, but sometimes we will have to appeal to different texts he authored, mainly in order to justify certain premises.

Although Aquinas’s proofs for the uniqueness of God are manifold, some of them are similar; so, we can distinguish the following types of arguments: (1) proofs in which Aquinas assumes the identity of essences of God-like beings (the first argument from Summa Theologiae, the third from Scriptum Super Libros Sententiarum and the ninth from Summa Contra Gentiles); next, (2) proofs in which he uses the principle of the identity of indiscernibles (whose formulation people usually ascribe to Leibniz—here the relevant arguments are: the second proof from Summa Theologiae, Scriptum Super Libros Sententiarum and Summa Contra Gentiles); (3) the proofs from Summa Contra Gentiles in which Aquinas uses the notion of specification (specificare) and designation (designare); (4) proofs in which he assumes that for every quality $P$ there is something that for every object having $P$ this thing is the unique cause of $P$ in them; (5) proofs where

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1 This article is a revised fragment of my master dissertation entitled Proofs for the uniqueness of God in the writings of St. Thomas Aquinas, supervised by Prof. Jerzy Perzanowski and defended at Nicholas Copernicus University in Torun in 2001. Portion of this paper has been published in Polish as Kakol 2006.
Aquinas assumes some kind of teleological order in the world; and (6) proofs in which the central notion is motion, in an Aristotelian sense. Not all the proofs are, of course, interesting from the logico-philosophical point of view. Hereafter I shall focus on the arguments of the first type.²

Now one should explain why we shall consider in the first case the proofs from S.th., though—as we know—this is the earliest of Aquinas’s works analyzed here. The rationale is that this is probably the most popular of Thomas’s texts, yet, on the other hand, the Angelic Doctor conceived it only as a concise manual. That is why S.th. alone cannot, as we will be able to see later, give us the full picture of Aquinas’s attacks on the problems we are interested in here. Other Aquinas’s texts are considered here according to their chronological order.

2. Arguments of the first type

Summa theologica
I, q. 11, a. 3
The first argument

Manifestum est enim quod illud unde aliquid singulare est hoc aliquid, nullo modo est multis communicabile. Illud enim unde Socrates est homo, multis communicari potest; sed id unde

For it is evident, that something, in virtue of which a given individual is the very particular thing, cannot be shared by many things in any way. Something, in virtue of which Socrates is a man, can

² The description of editions of Thomas’s writings is in the bibliography. Translation of Aquinas’s texts is done by the author, if not otherwise indicated. The translation base for Summa Theologiae is Marietti’s edition from 1894, hence, suitable texts are cited according to this edition. Thomas’s works are cited in the following way: Summa Theologiae is denoted by S.th.; Scriptum Super Libros Sententiarum is denoted by Scriptum; De Ente et Essentia is denoted by De ente; Summa Contra Gentiles is denoted by Scg. “Dist.” denotes distinction, “c.”- chapter (caput), “q.”- question, “a.”- article. In the case of S.th. only the first part (pars prima) of this work is used, whereas in the case of Scriptum and Scg – only the first book; therefore, we shall omit this information in the subsequent quotes. Next, we shall give page number, column number and line number. Example: S. th. q.1, a.1, p.15, col.2, l. 1-10 denotes Marietti’s edition of Summa theologiae from 1894, part one, question 1, article 1, page 15, column 2, lines 1-10.
As to the chronology of St. Thomas’s works, see Weisheipl 1983.
est hic homo, non potest communicari nisi uni tantum. Si ergo Socrates per id esset homo, per quod est hic homo, sicut non possunt esse plures Socrates, ita non possent esse plures homines. Hoc autem convenit Deo. Nam ipse Deus est sua natura, ut supra ostensum est (q.3, a.3). Secundum igitur idem est Deus et hic Deus. Impossibile est igitur esse plures deos.

(S.th. q.11, a.3, p.60, col.2, l.48-50 – s.61, col.1, l.1-12)

be shared by many, yet something, in virtue of which he is the very particular man, can pertain to but one thing. So if Socrates were a man in virtue of the same thing by which he is that very particular man, then, as it is not possible that many Socrateses exist, it would not be possible that many men could exist. Yet this pertains to God. For God is His own essence (q.3, a.3). So He is God in virtue of the same thing by which He is that particular God. Therefore, it is impossible that many Gods exist.

In q.3, a.3 Thomas argues that in God there is no difference between subject (individual, suppositum) and essence. He justifies it claiming that in God there is no composition of matter and form (q.3, a.2). Essence contains only that which pertains to the definition of the species, yet in particulars composed of matter and form there is individual matter along with accidents, which does not enter into definition, and hence, is not contained in essence. But it seems that it does not follow that pure form is essence provided that there are certain accidents which are independent of matter. Thomas writes about accidents which individuate matter (materia individualis cum accidentibus omnibus individuantibus ipsam - q. 3, a. 3, p. 19, col. 2, l. 2-4). Yet are there any accidents independent of matter? The text we consider does not answer the question. Even if God is His essence, it does not suffice for proving His uniqueness, for we would have to assume

3 In the explored texts Aquinas treats such terms as essentia, quidditas and natura as synonyms. In De ente c. 1, p.10, l.24-25 we read that essence is something “through which a thing pertains to its proper kind or species” (per quod res constituitur in proprio genere vel specie), and is this that is denoted by this thing’s definition. The problem is that if essence is defined independently of division of the universe into kinds and species, then it is hardly distinguishable from the proper accidents (accidentia propria). On the other hand, if – in order to define essence – we must have this division first, we are not allowed to make such division by appealing to essential properties.
that God-like beings have the same essence. Let ‘ess’ be a functional term; ‘ess(x)’ should be read as ‘x’s essence’. Let ‘D(x)’ denote ‘x is a God-like being’. That every such an object is its essence is expressed by:

\[ \forall x (D(x) \rightarrow \text{ess}(x) = x) \]

So there is at most one such object, if there is only one essence of such objects, i.e.

\[ \forall x \forall y (D(x) \land D(y) \rightarrow \text{ess}(x) = \text{ess}(y)) \]

A question arises: how to interpret formulae ‘\(\text{ess}(x) = x\)’ or ‘\(\text{ess}(x) = \text{ess}(y)\)’? The thinker familiar with the contemporary philosophy can easily be misled here. For example, Alvin Plantinga in his *Does God Have A Nature?* presents the following *reductio* of the thesis ‘God is His essence’: Essence is a set of certain properties, God is His essence, hence, God is a set of certain properties, which amounts to the claim that there is no God (Plantinga 1980, p. 47). But the proper conclusion is of course that Aquinas had something different in mind.

*Substance* is that which is not in something else as in a subject; its opposite is *accident*, which directly or indirectly is in substance. Another account is: P is x’s accident if and only if P is not included in x’s essence. x’s accidents can be either *necessary* for x (accidentia propria) or *contingent*; it is also said that the first ones “follow from essence” or that they have a “cause” in that essence.

From this description Aquinas concludes that God is His essence provided that there is no accident in Him. Therefore, the expression ‘*Deus est sua essentia*’ means simply that there is no accident in God.

This last claim is investigated by Aquinas in q.3, a.6 (“Are there any accidents in God?”). In the first argument Thomas holds that the relationship between *suppositum* and accident is an exemplification of the relationship between potency and act, whereas God is a pure act. In the second one he says that nothing can be superadded to pure existence, whereas God is a pure existence. The third one is:

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4 Although it is problematic at least in that the term *causa* has a very broad meaning here. I translate it consequently as a “cause”, but in many cases it should be understood as a “reason”.

Omne quod est per se, prius est eo quod est per accidentem. Unde cum Deus sit simpliciter primum ens, in eo nihil potest esse per accidentem. Sed nec accidentia per se in eo esse possunt; sicut risibile est per se accidentem hominis; quia huissusmodi accidentia causantur ex principiis subjecti. In Deo autem nihil potest esse causatum, cum sit causa prima. Unde relinquitur quod in Deo nullum sit accidentem. (S.th. q.3, a.6, p.22, col.2, l.31-41)

Everything, which is by itself, is prior to what is by accident. Hence, since God is by His own nature the prime being, nothing can be in Him by accident. And also proper accidents cannot be in Him, as, for example, being able to laugh is man’s proper accident; because an accident of that kind has its cause in the subject’s principles. Yet nothing, which is in God, can have any cause, since God is the first cause. Hence, it follows that there is no accident in God.

The crucial premise of that reasoning was formulated by Aquinas in q.3, a.4:

Quia quidquid est in aliquo, quod est praeter essentiam eius, oportet esse causatum: vel a principiis essentiae, sicut accidentia propria consequentia speciem; ut risibile consequit hominem, et causatur ex principiis essentialibus speciei: vel ab aliquo exteriori, sicut calor in aqua causatur ab igne. (S.th. q.3, a.4, p.20, col.1, l.42-49)

It is necessary that everything in something else, and outside of the essence of that in which it is, has a cause: either the principles of essence will be that cause, as proper accidents following from the species, as being able to laugh pertains to man, and the essential principles of the species are its cause, or something external to it will be that cause, as, e.g., fire is the cause of heat in water.

The principle stating that “what is outside of the essence, has a cause”, is not explained in S.th.; furthermore, that God does not have any cause, need not entail that nothing in God has any cause. As we shall see, more information related to this problem is in Scg.

Scriptum super libros Sententiarum
I, dist. II, q.1, a.1
The third argument

Praeterea, eius in quo non differunt suum esse et sua quidditas, non potest participari quidditas sua sive essentia, nisi et esse participetur. Sed quandocumque dividitur essentia alicuius per participationem, participatur essentia eadem secundum rationem et non secundum idem esse. Ergo impossibile est eius in quo non differet essentia et esse, essentialem participationem dividi vel multiplicari. Tale autem est Deus (...). (Scriptum I, dist.II, q.1, a.1, p.60, l.27-36)

What is more, the essence of the thing, in which there is no difference between the very thing’s essence and its existence, cannot be participated in, if the existence were not be participated in. Yet whenever one’s essence is divided by participation, the same essence according to its concept is participated in, but not according to its existence. So, it is impossible for something in which there is no difference between essence and existence to be divided by essential participation or multiplied. Yet God is a thing of that kind (...).

This argument seems to be very unclear. Let me reformulate it in the following way: essence is “divided by participation” if there are at least two beings having that essence; yet these beings differ from one another in their existence, so they participate in that essence as having the same essence, yet not as existing. Hence, there cannot be more beings having the same essence, if this essence does not differ from their existence. More formally, using symbols ‘ess’ and ‘D’ introduced above, and introducing term ‘ex(x)’ (‘x’s existence’), one can prove that there is at most one God-like being. The theorem that is to be proved is that of the uniqueness of God:

(!) \( \forall x \forall y (D(x) \land D(y) \rightarrow x = y) \)

The sentence “Sed quandocumque...” can be expressed by:

(3) \( \forall x \forall y (x \neq y \land ess(x) = ess(y) \rightarrow ex(x) \neq ex(y)) \)

Following tradition (in particular, the tradition of existential thomism), I usually translate esse as ‘existence’.
Different things, participating in the same essence, differ according to their existence.
Next, God is such that His essence is His existence, that is:

\[ \forall x \ (D(x) \rightarrow \text{ess}(x) = \text{ex}(x)) \]

One can prove that from (3) and (4), (!) does not follow. The proof will be valid if we add formula (2) to the premises, i.e.

\[ \forall x \forall y \ (D(x) \land D(y) \rightarrow \text{ess}(x) = \text{ess}(y)) \]

So we have the same assumption as we had in the first argument of *S.th.*, that is, that God-like objects have the same essence.\(^6\)

It is worth noticing that our interpretation of the sentence “*Sed quandocumque...*” can be seen as not coinciding with Aquinas’s intention. One could suggest the following, alternative account: if \(x\) and \(y\) participate in a certain essence, then they participate in the same essence according to its concept, but not in the same essence according to its existence, i.e. \(x\) and \(y\) have the same essence according to its concept, yet not according to its existence.

We can make use of the following analogy:

![Diagram of two circles, one covered and one uncovered, illustrating the concept of color.](image)

We can say that both circles have “the same color according to the uncovered color”. Assume that, in addition to that, we know that they have “different color according to the covered color”. It is easy to deduce from that that the following is true of at least one circle: the color covered is not identical to the color uncovered. Moreover, it is true of one circle that the

\(^6\) Formal proofs can be found in Appendix.
color of full circle is not identical to the color covered. Similarly, Thomas is right in concluding that it is true of some being that its essence is not identical to its existence.

But it seems that these phrases expressing relative identity can be eliminated, for they suggest that existence is a part (or an aspect) of essence; whereas Thomas in S.th., q.3,a.4 (Utrum in Deo sit idem essentia et esse) argues for theorem (4) in his first argument as follows: God’s existence cannot be outside of His essence, for otherwise either (i) it has to be “caused” by His essence, or (ii) it has to be caused by something external to essence. Importantly, Aquinas does not take into account here that existence could be part of essence.

*Summa Contra Gentiles*
I, c. 42
The ninth argument

*Adhuc, Si sunt duo dii, aut hoc nomen Deus de utroque praedicatur univoce aut aequivoce. Si aequivoce, hoc est praeter intentionem praesentem; nam nihil prohibet rem quamlibet quolibet nomine aequivoce nominari, si usus loquentium admittat. Si autem dicatur univoce, oportet quod de utroque praedicetur secundum unam rationem; et sic oportet quod in utroque sit una natura secundum rationem. Aut igitur haec natura est in utroque secundum unum esse, aut secundum aliud et aliud. Si secundum unum, ergo non erunt duo, sed unum tantum; duorum enim non est unum esse, si substantialiter distinguantur. Si autem est aliud et aliud esse in utroque, ergo neutrum erit sua quidditas vel suum esse; sed hoc*  

Next, if there are many gods, then either this name ‘God’ is predicated of these two univocally or equivocally. If equivocally, it is beyond our current intention: since nothing prevents any thing being named by any name, provided that this is allowed by the speaker’s habit. But if it is predicated univocally, it is necessary that it is predicated of these two according to the same concept, and thus, one nature according to the concept must be in them. So either this nature is in these two according to one existence, or not. If the former is the case, then there are no two gods but only one: the existence of two ones is not one, if they would differ according to substance. Yet if the existence in them is different in each case, neither of them would
be His own essence or His existence; yet this should be assumed about God, as it has been demonstrated. Hence, neither of them is something we mean by God. Thus, it is impossible to assume two gods.

The premises of Thomas’s reasoning are: “Aut igitur haec natura est in utroque secundum unum esse, aut...”, i.e.

(5) $\forall x \forall y \ (D(x) \land D(y) \rightarrow ((\text{ess}(x) = \text{ess}(y) \land \text{ex}(x) = \text{ex}(y)) \lor (\text{ess}(x) = \text{ess}(y) \land \text{ex}(x) \neq \text{ex}(y))))$

“Si secundum unum...”, i.e.,

(6) $\forall x \forall y \ (\text{ess}(x) = \text{ess}(y) \land \text{ex}(x) = \text{ex}(y) \rightarrow x = y)$

“duorum enim...” is a version of the principle of the one-to-one correlation between being and its existence, i.e.

(7) $\forall x \forall y \ (x \neq y \rightarrow \text{ex}(x) \neq \text{ex}(y))$

Our interpretation of (7) may raise certain doubts. Should not identity of two objects mean that they differ according to substance? The answer depends of course on what is the meaning of the term 'substantialiter’ here. It
seems that Thomas means here that we have two substances (i.e. two individuals). If we read the sentence as *duorum enim non est unum esse, si essentialiter distinguantur*, then proofs for the uniqueness of God would be simply unintelligible. Secondly, Aquinas assumes (7) in Scg: the evidence is in such text as *habet enim res unaquaeque in seipsa esse proprium ab omnibus aliis distinctum* (Scg, c.14, p.15, col.1, l.20-22 – “For every thing has in itself its own existence, different from any other [existences]”) or *Esse proprium cuiuslibet rei est tantum unum* (Scg c.42, p.40, col.1, l.1-2 – “Existence proper to every thing is only one”).

“*Si autem...*” is not clear: the consequent of this sentence can mean “*ergo neutrum erit sua quidditas, aut neutrum erit suum esse*”, i.e.

\[ \forall x \forall y (\text{ess}(x) = \text{ess}(y) \land \text{ex}(x) \neq \text{ex}(y)) \rightarrow \\
(x \neq \text{ess}(x) \land y \neq \text{ess}(y)) \lor (x \neq \text{ex}(x) \land y \neq \text{ex}(y)) \]

It seems that Thomas treats this formula as a logical theorem\(^7\), in this case—speaking in a modern manner—the conclusion follows from the elementary theory of identity. But in fact it is not a logical theorem. Thus, the consequent could be then read as “*ergo neutrum erit aut sua quidditas, aut suum esse*”, i.e.,

\[ \forall x \forall y (\text{ess}(x) = \text{ess}(y) \land \text{ex}(x) \neq \text{ex}(y)) \rightarrow \\
(x \neq \text{ess}(x) \lor x \neq \text{ex}(x)) \land (y \neq \text{ess}(y) \lor y \neq \text{ex}(y)) \]

Unfortunately, this also is not a logical theorem. So let us try this way: the consequent will be “*ergo neutrum sua quidditas erit suum esse*”, i.e.

\[ \forall x \forall y (\text{ess}(x) = \text{ess}(y) \land \text{ex}(x) \neq \text{ex}(y)) \rightarrow (\text{ess}(x) \neq \text{ex}(x) \land \text{ess}(y) \neq \text{ex}(y)) \]

But this is not a logical theorem, either. We can read the consequent as speaking of some of them: so “*neutrum*” is wrong here.

\[ (8) \forall x \forall y (\text{ess}(x) = \text{ess}(y) \land \text{ex}(x) \neq \text{ex}(y) \rightarrow (\text{ess}(x) \neq \text{ex}(x) \lor \text{ess}(y) \neq \text{ex}(y))) \]

\(^7\) Strictly speaking, a theorem of logic enriched by constants ‘ess’ and ‘ex’.
It is easy to demonstrate that that this is a theorem of the elementary theory of identity (enriched by constants ‘ess’ and ‘ex’). It can also be demonstrated that from (5), (7) and (8), (!) does not follow, unless we add premise (4). So Thomas himself adds this in the argument we are considering.

3. Comparison and further analysis

From what we have said so far, it follows that we have the following proofs of the first type:

\[ S.th.: \ (1), (2) \vdash (!) \]
\[ Scriptum: (2), (3), (4) \vdash (!) \]
\[ Scg: \ (4), (5), (7), (8) \vdash (!) \]

We can ask which argument is based on the weakest premises. Now, one can demonstrate that (5) is equivalent to (2) and that (7) implies (3). So (2) is a premise of all three proofs, and the argument from \textit{Scriptum} is better than the argument from \textit{Scg}.

Recall that in the comments on the first proof from \textit{S.th.} we have said that lack of accidents in a being entails, according to Aquinas, that this being is its essence. It is worth considering this issue for the following reason: since all three proofs are based on the theorem stating that essences of God-like beings are identical, one should prefer the proof containing the smallest number of premises, i.e. the first proof from \textit{S.th}. In other words, assuming that essences of God-like beings are identical, it suffices to prove that every such object is its own essence.

\textit{Deus est sua essentia}

Hereafter, we shall try to analyze certain arguments form chapter 23 of \textit{Scg} ("That in God there are no accidents").

One could ask why we focus on chapter 23 and not on chapter 21 ("That God is His essence"). Now, in the latter Thomas gives us five arguments for the theorem that God is his essence. Yet the fifth is very unclear; in the fourth Aquinas allows himself to refer at most to chapter 13, but it is insufficient since chapter 13 is devoted only to the causes of motion in the Aristotelian sense; the third is the counterpart of the one from \textit{S.th}. q.3, a.3 we have already quoted; the second refers to chapter 23 (which we will com-
ment on below); so there remains the first which refers to chapter 18 ("That in God there is no composition").

*Quod in Deo nulla sit compositio* (Scg, c.18).

This chapter consists of seven arguments. The third and the fourth are unclear:

*Omne compositum est potentia dissolubile, quantum est ex ratione compositionis, licet in quibusdam sit aliquid aliud dissolutioni repugnans. Quod autem est dissolubile est in potentia ad non-esse; quod Deo non competit, cum sit per se necesse esse. Non est ergo in eo aliqua compositio.*

*Amplius, omnis compositio indiget aliquo componente; si enim compositio est, ex pluribus est. Quae autem secundum se sunt plura, in unum non conveniunt, nisi ab aliquo componente uniantur. Si igitur compositus esset Deus, haberet componentem; non enim ipse seipsum componere posset, quia nihil est causa suiipsius; esset enim prius seipso, quod est impossibile. Componens autem est causa efficiens compositi. Ergo Deus haberet causam efficientem; et sic non esset causa prima, quod supra habitum est.*

([The third argument] Every composition is potentially decomposable, if it exists in virtue of being composed, although in certain things there is something else, which resists decomposition in them. What is decomposable, is potentially not existent, which does not pertain to God, since He is by himself necessary being [or: it is necessary by itself that He exists]. Hence, there is no composition in Him.

[The fourth argument] Similarly, every composition requires something composing it, for if there is composition, it is such in virtue of the many. Now, things that are by themselves numerous, would not have been unified if they had not been unified by something unifying them. So, if God were composed, He would have something composing Him, for He could not compose Himself, as nothing is the cause of itself: otherwise He would exist before Himself, which is impossible. And a thing composing is the efficient cause of a thing composed. Hence, God would have an efficient cause, as
a result of which he would not be the first cause, for which we have agreed above.

The first argument is also unclear. The fifth is may be summarized as follows. The more simple a thing is, the more perfect it is. Cause is more perfect then its effect. God does not have any cause; hence, He is the most perfect being. *Ergo*, God is the simplest being. This argument fails for Thomas proved neither that God is the cause of everything nor that he is unique. Assuming that proofs for the existence of God in chapter 13 are sound, he at best demonstrated that He has no cause. For example, let domain D = \{a, b, c\} be given; and let a be a cause of c and b be a cause of c as well. We have here two God-like beings, but from the principle “cause is more perfect then its effect” alone we cannot conclude which God-like being is more perfect.

The sixth argument has, in turn, unclear premises: God is the most perfect being. In composites, parts are imperfect as compared to the whole, therefore, such things cannot be the most perfect beings. *Ergo*, God is not a composite. So, let us take a look at the second and the seventh argument:

*Item, omne compositum posterius est suis componentibus. Primum ergo ens, quod Deus est, ex nullis compositum est.* (Scg c.18, p.17, col.2, l.4-6)

[c. 18, The second argument]

Also, every composite is posterior to its components. So the first being, which is God, is not composed from anything.

Words ‘primum’, ‘posterius’ suggest some kind of order (not necessarily a linear one, though the text suggests so). Is this a causal order (“the first” would mean here “without any cause”)? It seems not: otherwise we would liken it to the fourth argument from chapter 18, which we have already quoted.

*Item, ante multitudinem oportet invenire unitatem. In omni autem composito est multitudo. Igitur oportet id, quod est ante omnia, scilicet Deum, omni compositione carere.* (Scg c.18, p.17, col.2,)

[c. 18, The seventh argument] Moreover, before each multiplicity one should find unity. But in every composite is multiplicity. So it is necessary that this, which is before everything, namely God, is
The expression ‘ante omnia’ entails uniqueness. Yet, interestingly, the parallel text from *S.th*. q.3, a.7, p.23, col.1, l.44-45 throws some light on the second argument: “omne compositum est posterius suis componentibus, et dependens ex eis”. What is dependence (dependentia)? Recall the last argument from chapter 23. “Substance does not depend on accidents, though accidents depend on substance. But a thing that does not depend on something else, sometimes can be discovered as being without that other thing”—substantia non dependet ab accidente, quamvis accidens dependeat a substantia. Quod autem non dependet ab aliquo potest aliquando inveniri sine illo—Scg c.23, p.25, col.1, l.39-43. It follows that if *x* does not depend on *y*, than *x* can exist without *y*; and, probably, if *x* depends on *y*, than *x* cannot exist without *y*. From this provisional explication of the notion of dependence we can conclude that Thomas holds:

\[(9) \forall x \ (x \text{ is composed } \rightarrow \forall y \ (y \text{ is a component of } x \rightarrow x \text{ cannot exist without } y)\]

and

\[(10) \forall x \ (D(x) \rightarrow \neg \exists y \ (y \neq x \wedge x \text{ cannot exist without } y))\]

i.e.

\[\forall x \ (D(x) \rightarrow \forall y \ (x \text{ cannot exist without } y \rightarrow x = y))\]

where ‘D(x)’ denotes ‘*x* is a God-like being’.

The problem lies in that if Aquinas substitutes accidents for *y*, then in the case of improper accidents, theorem (9) is false: if I have a beard, then—since this is for me a contingent accident—I can exist without my beard. Now we have to turn to the chapter directly devoted to the question concerning accidents.

*Quod in Deo non sit accidens*
Not all the proofs from this chapter deserve our attention. Certain versions of the first and the third have already been presented (*S.th.* q.3, a.6; see the section devoted to *S. th.*). The sixth appeals to simplicity. The fifth assumes, *inter alia*, that God is the cause of everything. God has no accidents; hence, He is his essence, since being such is more perfect than if He were not His essence. Cause is more perfect than its effect. God is the cause of everything. Ergo, God has no accidents. So let us analyze the fourth and the second argument.

The fourth argument from c.23

_Cuique inest aliquid accidentaliter, est aliquo modo, secundum suam naturam, mutabile; accidens enim de se natum est inesse et non inesse. Si igitur Deus habet aliquid accidentaliter sibi conveniens, sequetur quod ipse sit mutabilis; cuius contrarium demonstratum est. (Scg c.23, p.25, col.1, l.14-21)_

Everything in which there is something accidentally, is somehow by its nature changeable; for accident is by its nature being in and not being in [i.e. by its nature it has potentiality for being in something, and also for not being in something]. Hence, if God has something accidentally, it follows that He is changeable; whose opposition has been demonstrated above.

The sentence “*accidens enim*” can be interpreted as follows:

(P)  \[ \forall y \forall x \ (y \text{ is an accident of } x \leftrightarrow x \text{ can be } y \land x \text{ may not be } y) \]

Keeping in mind the characterization of accidents from the section devoted to *S. th.*, it is obvious that (P) is about improper accidents only. The “changeability” of \(x\) consists in that \(x\) can be yet may not be \(y\)—we can call it “modal changeability”. I think it is worth referring here to Knuuttila’s observation that in medieval philosophy (Aquinas included) the so called statistical interpretation of modality was favored, and, in particular, the famous “Principle of Plenitude”.\(^8\) This explains Thomas’s reasoning, particularly that of his “Third Way”. The principle of plenitude can be formulated thus:

\(^8\) Knuuttila 1982, 342-357.
∀x∀y (x can be y/ x may not be y → ∃t (x is y at t /∃ t’ x is not y at t'))

Our interpretation needs some comment. The expression ‘x can be y’ is connected with the non-modal expression ‘x is y’ (or ‘x is y at t’). This formula can be interpreted, of course, in many ways. Since the chapter under consideration has to do with accidents, assume that ‘is’ expresses here a certain relation holding between a substance and accidents. In order to avoid introducing variables of different kinds, we will interpret ‘x is y’ as ‘accident y pertains to x’. In addition, we will assume that this relation is asymmetric. Recall also that (P) is the definition of improper accident. Let me add that alternative formulations of the principle of plenitude are also allowed.

In chapter 13 Thomas tries to demonstrate that God, i.e., *primus motor separatus, omnino immobils*, exists. Of course, one should ask what kind of *motus* is taken into account here. The answer is: at least each kind of physical motion in the Aristotelian sense, hence, e.g., if this accident is a quality y, then change from not-y to y is a kind of motion (*...ut probat, inducendo in singulis speciebus motus.*, *)si autem secundum aliam speciem motus moveatur... etc.* [Scg c.13, p.12, col.1, l.33-34; p.13, col.1, l.7-9; emphasis – TK)].

Now we can prove that there are no improper accidents in God.

Let ‘xMEy’ denote ‘x can be y’
‘xM-Ey’ denotes ‘x may not be y’
‘xEy’ denotes ‘x is y’
‘x-Ey’ denotes ‘x is not y’
‘xEyt’ denotes ‘x is y at t’
‘x-Eyt’ denotes ‘x is not y at t’
‘yPx’ denotes ‘y is x’s improper accident’
‘D(x)’ denotes ‘x is a God-like being’.

One must add that the proof of the theorem that improper accidents do not pertain to God should be based on the basic modal logic of the expressions ‘x can be y/ x may not be y’. We will not, however, search for such a logic in Aquinas’s writings: instead, we will simply treat ME, M-E, E, -E as primitive predicates.

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9 See, for example, Perzanowski 1993, 10-16.
Apart from the definition of $P$

\[(\text{def. of } P) \quad \forall x \forall y \ (yPx \leftrightarrow x \text{ME} y \land x \text{M-E} y)\]

we assume that

\[(\text{God’s immobilitas}) \quad \forall x \forall y \ (D(x) \rightarrow \neg (\exists t \ xEy \land \exists t' \ x-Ey'))\]

Our formalization assumes that the principle of plenitude consists of two propositions:

\[(\text{Positive principle of plenitude}) \quad \forall x \forall y \ (x \text{ME} y \rightarrow \exists t \ xEy)\]

\[(\text{Negative principle of plenitude}) \quad \forall x \forall y \ (x \text{M-E} y \rightarrow \exists t' \ x-Ey')\]

They entail:

\[(11) \quad \forall x \forall y \ (x \text{ME} y \land x \text{M-E} y \rightarrow \exists t \ xEy \land \exists t' x-Ey')\]

From (def. of $P$), (God’s immobilitas) and (11) it follows that

\[(12) \quad \forall x \ (D(x) \rightarrow \neg \exists y \ yPx)\]

Now we turn to the second argument that there are no accidents in God.

The second argument from c.23

\textit{Amplius, Omne quod inest alicui accidentaliter, habet causam quare insit, cum sit praeter essentialiam eius cui inest. Si igitur aliquid accidentaliter sit in Deo, oportet quod hoc sit per aliquam causam. Aut ergo causa accidentis est ipsa divina substantia, aut aliquid aliud. — Si}

Similarly, everything being in something accidentally has a cause in virtue of which it is in that thing, since it is outside of that thing’s essence. So if there were something in God accidentally, it would have to be so in virtue of some cause. And either the divine substance itself is such
a cause or something else. If something else is such a cause, it is necessary that something else acts upon the divine substance. For nothing can introduce any form—either substantial form or accidental—into some being receiving that form, if not acting upon that thing in some way, for to act is nothing else than to make something actual, and this process is done through some form. Therefore, God will be acted upon and moved by some agent, contrary to what has been established (in chapter 13).

In the comments on the first argument from S.th. we said that the principle ‘what is outside of essence, has a cause’ was not explained. Yet, if we refer to chapter 15 we can agree that Thomas assumes—regarding improper accidents—the theorem analogous to a certain proposition from chapter 15. Which proposition? First let us look at the following text from this chapter (this is a fragment of the fourth argument for the eternity of God):

\begin{itemize}
  \item[(1)] Videmus in mundo quaedam quae sunt possibilia esse et non esse, scilicet generabilia et corruptabilia.
  \item[(2)] Omne autem quod est possibile esse, causam habet;
  \item[(3)] quia, cum de se aequaliter se habeat ad duo, scilicet esse et non esse, oportet, si ei approprietur esse, quod hoc sit ex aliqua causa.
\end{itemize}

(Scg. c.15, p.15, col.2, 1.42-49)
Sentence (1) is not unproblematic: we cannot exclude that—similarly as it is e.g. in sentence (2)—it says about *de re* logical modality (as opposed to ontological modality). Let ‘E(x)’ denote ‘x exists’, ‘ME(x)’ denote ‘x is a possible being’, ‘yCAUx’ denote ‘y is a cause of x’, ‘◊’ denote ‘it is possible that’. Sentence (2) can be formalized as:

$$\forall x (\Diamond E(x) \rightarrow \exists y \ y \ CAUx)$$

or as:

$$\forall x (ME(x) \rightarrow \exists y \ y \ CAUx)$$

But this does not seem to be true. Thomas in sentence (3) adds that having a cause holds in case of existing things. What is something which ‘in itself refers equally to existence and to non-existence’? It seems that this expression should be understood as follows: this thing can exist and may not exist (or: it is possible that it exists, and it is possible that it does not exist). This characterization is just the characterization of *contingent* being, and such modality occurs in sentence (2) under somewhat misleading name “*possible esse*”. Let ‘ME(x)’ not denote ‘x is a possible being’, and ‘x can exist’; let ‘M-E(x)’ denote ‘x may not exist’; ‘P(x)’ denote ‘x is contingent’. The definition of contingency will be:

$$P(x) := ME(x) \land M-E(x)$$

or:

$$P(x) := \Diamond E(x) \land \Diamond \neg E(x)$$

Sentence (3) says that what is contingent and exists, has a cause:

$$(i) \quad \forall x (P(x) \land E(x) \rightarrow \exists y \ y \ CAUx)$$

This proposition gives us clue: As far as improper accidents are concerned, assume that

$$\forall x \forall y (x \ can \ be \ y \land x \ may \ not \ be \ y \land x \ is \ y \rightarrow \exists z \ z \ is \ a \ cause \ of \ y \ in \ x)$$

Formally,
The problem of transition from the sentence ‘God has no cause’ to the one ‘nothing in God has any cause’ remains. It is worth noticing that in the argument we analyze, Aquinas makes use of a completely different premise, namely, that occurrence of the relation of causality entails occurrence of change in the object upon which this cause acts. In order to make this proposition more similar to (13), we formalize it as:

(14) \( \forall x \forall y \forall z \left( z \text{CAU}yx \rightarrow \exists t \left( xEyt \land \exists t' \left( t \neq t' \land x\text{-E}yt' \right) \right) \right) \)

Moreover, as we can see, from (13) and (14) we have that something, which can be \( y \) and may not be \( y \), and is \( y \), sometimes is not \( y \)!

(15) \( \forall x \forall y \left( x\text{M}Ey \land x\text{M-E}y \land xEy \rightarrow \exists t \ x\text{-E}yt \right) \)

Although theorem (15) is not identical with the negative principle of plenitude (more precisely, it follows from this principle), it seems that it expresses a similar conception of modality to the one expressed in the fourth argument of chapter 23. Therefore, we do not have a proof for the lack of improper accidents in God, the proof of which can be independent of such a nonstandard interpretation of modality.

Another proof for the lack of improper accidents in God can be made using (13), (14), (def. of P) and (God’s immobilitas). It should be pointed out here that the proposition that is to be proved will differ from the one that has been given above (see the section devoted to the fourth argument from c. 23), namely (12):

\( \forall x \forall y \left( D(x) \rightarrow \neg yPx \right) \)

The theorem that is to be proved will be:

(16) \( \forall x \forall y \left( D(x) \rightarrow \neg(yPx \land xEy) \right) \)

These two formulae have very similar senses: the latter says that an allegedly God-like being’s accidents are not improper ones, whereas the former says that nothing is a God-like being’s accident.
So far we have taken into account improper accidents. What about the proper ones? The second part of the second argument from chapter 23 is:

Si autem ipsa divina substantia est causa accidentis quod sibi inest; impossibile autem est quod sit causa illius, secundum quod est recipiens ipsum; quia sic idem secundum idem, faceret seipsum in actu; ergo oportet, si in Deo est aliquod accidentis, quod secundum aliud et aliud recipiat et causet accidens illud; sicut corporalia recipiunt propria accidentia per naturam materiae, et causant per formam. Sic igitur Deus erit compositus; cuius contrarium superius probatum est. (Scg c.23, p.24, col.2, l.48-55; p.25, col.1, l.1-5)

If the divine substance itself is the cause of accidents which are in this substance; it is impossible for this substance to be the cause of this accident since it itself receives this accident, and then the same and under the same aspect would make itself actual. Hence, it is necessary—if there is any accident in God—that this substance under one aspect receives this accident, and under another causes this accident, as, for example, corporeal beings receive proper accidents through the nature of matter, and cause them through form. Thus, God will be composite; whose opposition has been demonstrated above.

The problem is that Aquinas seems to assume that there are proper accidents of certain immaterial substances, i.e. angels. In De ente c. 6 he writes:

...et quia in istis substantiis quidditas non est idem quod esse, ideo sunt ordinabiles in praedicamento; et propter hoc invenitur in eis genus, species et differentia, quamvis earum differentiae propriae nobis occultae sint (...). Accidentia autem propria substantiarum immaterialium nobis ignota sunt...  

...and since in these substances [i.e. immaterial ones] their essence is not the same as their existence, we can place them in the order of categories, and in virtue of this we discover in them kind, species and difference, though we do not know what their proper differences look like (...). We do not know the proper acci...
immaterialium nobis ignota sunt, unde differentiae earum nec per se nec per accidentales differentias nobis significari possunt. (De ente c.6, p.28, l.31-35; p.29, l.3-6)

That their proper accidents are not known by us does not mean that they do not exist. According to Thomas, angels are not composed of matter and form. But the solution is simple: proper accidents “follow” from nature, hence, they cannot individuate angels within one species – in other words, if they were different proper accidents, we would have different natures.

4. Summary

We have not gained a sound argument that there are no accidents in God, i.e., that God is His essence. Worse, Aquinas seems to take it for certain that God-like beings have the same essence. Therefore, the arguments we have called the arguments of the first type, seem very problematic.

5. Appendix

Formal proofs

The proofs will be presented according to the following scheme. First, (i) via dictum de omni, we omit (all or some) general prefix quantifiers, then (ii) using metatheorem (M) we deduce from such open formulas the theorem which is to be proved, and which will be also an open formula, and finally (iii) we use the rule of generalization. Since the relation of deducibility is transitive, from premises in a closed form, we obtain the theorem that is to be proved.

Now the aforementioned metatheorem (M) is:

If $z_1,...,z_n$ are all the free variables (different from each other) in the formula $A$, and $b_1,...,b_n$ are different individual terms which do not occur in
formulas from set \( X \cup \{A, B\} \), and \( B(z_1/b_1, \ldots, z_n/b_n) \in Cn_{CPC-I}(X \cup \{A(z_1/b_1, \ldots, z_n/b_n)\}) \),
then
\[ A \rightarrow B \in Cn_{CPC-I}(X) \]

Where \( Cn_{CPC-I} \) is the consequence operator of (classic) first-order predicate calculus.
For brevity’s sake, we omit in proofs part (i) and (iii), and we will write simply about substitutions (of individual terms), but such substitutions are indeed made on open counterparts of premises.

\textit{Ad Scriptum:}

We prove that if (2), (3) and (4) are the case, so is (!).

1. \( D(a) \land D(b) \) (assumption, substitution)
2. \( a \neq b \land ess(a) = ess(b) \rightarrow ex(a) \neq ex(b) ((3), substitution) \)
3. \( ess(a) = ess(b) \land ex(a) = ex(b) \rightarrow a = b \) (2, classic propositional calculus [= CPC])
4. \( D(a) \rightarrow ess(a) = ex(a) \) ((4), substitution)
5. \( D(b) \rightarrow ess(b) = ex(b) \) (as above)
6. \( ess(a) = ex(a) \land ess(b) = ex(b) \) (1, 4, 5, CPC)
7. \( ess(a) = ess(b) \rightarrow a = b \) (3, 6, the rule of replacement of identicals, CPC)
8. \( D(a) \land D(b) \rightarrow ess(a) = ess(b) \) ((2), substitution)
9. \( a = b \) (1, 8, 9, CPC)

\textit{Ad Summa contra gentiles I, c. 42, the ninth argument:}

First we prove by \textit{reductio} that (8) is the case:

1. \( ess(a) = ess(b) \land ex(a) \neq ex(b) \) (assumption, substitution)
2. \( \neg(ess(a) \neq ex(a) \lor ess(b) \neq ex(b)) \) (assumption, substitution)
3. \( ess(a) = ex(a) \land ess(b) = ex(b) \) (2, CPC)

\textsuperscript{10} See Batog, 1994, 164.
4. \( \text{ess}(a) = \text{ess}(b) \land \text{ess}(a) \neq \text{ess}(b) \) \quad (1, 3, the rule of replacement of identicals)

We obtain a contradiction, hence—since (apart from CPC) we use only the rule of replacement of identicals—(8) is a theorem of the elementary theory of identity.

Now we prove (!) on the basis of (4), (5), (7) and (8):

Proof:
1. \( D(a) \land D(b) \rightarrow ((\text{ess}(a) = \text{ess}(b) \land \text{ex}(a) = \text{ex}(b)) \lor (\text{ess}(a) = \text{ess}(b) \land \text{ex}(a) \neq \text{ex}(b))) \) \quad ((5), substitution)
2. \( D(a) \rightarrow \text{ess}(a) = \text{ex}(a) \) \quad ((4), substitution)
3. \( D(b) \rightarrow \text{ess}(b) = \text{ex}(b) \) \quad (as above)
4. \( a \neq b \rightarrow \text{ex}(a) \neq \text{ex}(b) \) \quad (7), substitution)
5. \( D(a) \land D(b) \) \quad (assumption)
6. \( \text{ex}(a) = \text{ex}(b) \rightarrow a = b \) \quad (4, CPC)
7. \( (\text{ess}(a) = \text{ess}(b) \land \text{ex}(a) = \text{ex}(b)) \lor (\text{ess}(a) = \text{ess}(b) \land \text{ex}(a) \neq \text{ex}(b)) \) \quad (1, 5, modus ponens)
8. \( \text{ess}(a) = \text{ess}(b) \land \text{ex}(a) \neq \text{ex}(b) \rightarrow \\
(\text{ess}(a) \neq \text{ex}(a) \lor \text{ess}(b) \neq \text{ex}(b)) \) \quad (8), substitution)
9. \( \text{ess}(a) = \text{ex}(a) \land \text{ess}(b) = \text{ex}(b) \rightarrow \\
\neg (\text{ess}(a) = \text{ess}(b) \land \text{ex}(a) \neq \text{ex}(b)) \) \quad (8, CPC)
10. \( \text{ess}(a) = \text{ex}(a) \land \text{ess}(b) = \text{ex}(b) \) \quad (5, 2, 3, CPC)
11. \( \neg (\text{ess}(a) = \text{ess}(b) \land \text{ex}(a) \neq \text{ex}(b)) \) \quad (9,10, modus ponens)
12. \( \text{ess}(a) = \text{ess}(b) \land \text{ex}(a) = \text{ex}(b) \) \quad (7,11, CPC)
13. \( \text{ex}(a) = \text{ex}(b) \) \quad (12, CPC)
14. \( a = b \) \quad (13, 6, modus ponens)

Notice here that (8) is in fact redundant, since one can make a much simpler proof of (!) in virtue of (5)’s being equivalent to (2)—we shall omit the simple proof for such equivalence.

Ad The fourth argument from c. 23:

From (def. of P), (God’s immobilitas) and (11) it follows that

\[ \forall x (D(x) \rightarrow \neg \exists y yPx) \]
From classic predicate calculus, this formula is equivalent to:

$$\forall x \forall y (D(x) \rightarrow \neg yPx)$$

Thus, the proof will be as follows:

1. $bPa \leftrightarrow aM Eb \land aM Eb$ (def. of P)
2. $aM Eb \land aM Eb \rightarrow \exists t aEb t \land \exists t' aE b t'$ ((11), substitution)
3. $D(a) \rightarrow \neg(\exists t aEb t \land \exists t' aE b t')$ (God’s immobilitas, substitution)
4. $D(a)$ (assumption)
5. $\neg(\exists t aEb t \land \exists t' aE b t')$ (3, 4, modus ponens)
6. $bPa \rightarrow \exists t aEb t \land \exists t' aE b t'$ (1, 2, CPC)
7. $\neg bPa$ (5, 6, CPC)

Ad The second argument from c. 23:

We prove that (15) follows from (13) and (14):

1. $\forall z (zCAU ba \rightarrow \exists t (aEb t \land \exists t' (t \neq t' \land aE b t'))$ ((14), substitution)
2. $aM Eb \land aM Eb \land aE b \rightarrow \exists z zCAU ba$ ((13), substitution)
3. $aM Eb \land aM Eb \land aE b$ (assumption)
4. $\exists z zCAU ba$ (2, 3, modus ponens)
5. $\forall z (zCAU ba \rightarrow \exists t (aEb t \land \exists t' (t \neq t' \land aE b t')) \rightarrow$
   $(\exists z zCAU ba \rightarrow \exists t (aEb t \land \exists t' (t \neq t' \land aE b t')))$ (CPC-I)
6. $\exists t (aE b t \land \exists t' (t \neq t' \land aE b t'))$ (1, 4, 5, modus ponens)
7. $\exists t (aE b t \land \exists t' (t \neq t' \land aE b t'))$ (1, 4, 5, modus ponens)
8. $\exists t aE b t \land \exists t' (t \neq t') \land \exists t' aE b t' \rightarrow \exists t' aE b t'$ (CPC)
9. $\exists t' aE b t'$ (6, 7, 8, CPC, modus ponens)

Finally, we prove (16) on the basis of (13), (14), (def. of P) and (God’s immobilitas):

1. $\forall z (zCAU ba \rightarrow \exists t (aEb t \land \exists t' (t \neq t' \land aE b t'))$ ((14), substitution)
2. \( a\text{ME}b \land a\text{M Eb} \land a\text{Eb} \rightarrow \exists \ z \ z\text{CAU} ba \) ((13), substitution)

3. \( bPa \leftrightarrow a\text{ME}b \land a\text{M Eb} \) (def. of P, substitution)

4. \( D(a) \rightarrow \neg( \exists \ t \ a\text{Ebt} \land \exists \ t' a\text{-Ebt'}) \) (God’s \textit{immobilitas}, substitution)

5. \( D(a) \) (assumption)

6. \( \neg( \exists \ t \ a\text{Ebt} \land \exists \ t' a\text{-Ebt'}) \) (4, 5, \textit{modus ponens})

7. \( bPa \land a\text{Eb} \rightarrow \exists \ z \ z\text{CAU}ba \) (2, 3, CPC)

8. \( \forall \ z \ ( z\text{CAU} ba \rightarrow \exists \ t \ ( a\text{Ebt} \land \exists \ t'(t \neq t' \land a\text{-Ebt'}))) \rightarrow \)
\( \exists \ t \ ( z\text{CAU} ba \rightarrow \exists \ t \ ( a\text{Ebt} \land \exists \ t'(t \neq t' \land a\text{-Ebt'}))) \) (CPC-I)

9. \( \exists \ t \ ( a\text{Ebt} \land \exists \ t'(t \neq t' \land a\text{-Ebt'})) \rightarrow \)
\( \exists \ t \ a\text{Ebt} \land \exists \ t'(t \neq t') \land \exists \ t' a\text{-Ebt'} \) (CPC-I)

10. \( \exists \ t \ a\text{Ebt} \land \exists \ t, t'(t \neq t') \land \exists \ t' a\text{-Ebt'} \rightarrow \)
\( \exists \ t \ a\text{Ebt} \land \exists \ t' a\text{-Ebt'} \) (CPC)

11. \( \exists \ z \ z\text{CAU} ba \rightarrow \exists \ t \ ( a\text{Ebt} \land \exists \ t'(t \neq t' \land a\text{-Ebt'})) \) (1, 8, \textit{modus ponens})

12. \( bPa \land a\text{Eb} \rightarrow \exists \ t \ a\text{Ebt} \land \exists \ t' a\text{-Ebt'} \) (7, 9, 10, 11, \textit{modus ponens})

13. \( \neg( bPa \land a\text{Eb}) \) (6, 12, CPC)

**REFERENCES**

Original texts of St. Thomas:

\textit{De ente et essentia}, Taurini: Marietti MCMXXVI

\textit{Scriptum super libros Sententiarum Magistri Petri Lombardi episcopi parisiensis},

\textit{Summa contra gentiles seu de veritate catholicae fidei}, Taurini: Marietti MCMXXXVIII

\textit{Summa theologica}, diligenter emendata de Rubeis, Billuart et aliorum notis selectis ornata, Pars 1\textsuperscript{a}, t.I, Taurini: Marietti 1894\textsuperscript{6}


Subsidiary literature cited:


Kakol, T. 2006 „Analiza formalna wybranych dowodów na jedynosc Boga u swietego Tomasza z Akwinu”, \textit{Studia Philosophiae Christianae}, 1, 144-154.

