On the Problem of Defining the Present in Special Relativity: A Challenge for Tense Logic

THOMAS MÜLLER, BONN*

1. Introduction

According to our commonsense view, time can be divided up into the past, the present, and the future. The present (“the now”), separating the past from the future, plays a special role in this picture, and our commonsense view accordingly affords the present a distinguished metaphysical status: presentism, which is arguably the commonsense metaphysics of time, holds that what exists at any given time is exactly that which is present at that time.

At any rate, the present plays a central role in language: In English and in other Indo-European languages, all finite verb forms carry tense, an indexical temporal determination relative to the present. Tense logic, developed by Prior starting in the 1950ies, maps the temporal determinations of natural language onto a formal, modal-logical calculus. The project of tense logic is connected with two claims. The first is expressive adequacy: the tense-logical calculus claims to represent adequately the temporal distinctions present in natural language. The second is metaphysical adequacy: tense logic is often connected with the metaphysical doctrine of presentism mentioned above. This was certainly Prior’s own view when he wrote that “the present simply is the real considered in relation to two particular species of unreality, namely the past and the future” (Prior 1970, 245).

Special relativity (SR), the physical theory formulated by Einstein in 1905, has often been viewed as a challenge to our commonsense views of both space and time. Minkowski, in giving his famous geometrical interpretation of space-time (1908), proclaimed that the distinction between space and time must fall, and many philosophers have followed suit. For example, Quine in Word and Object remarks that “Einstein’s relativity principle […]

* Thanks to the audience in Kirchberg for fruitful discussion, and to Cord Friebe for detailed comments on a previous draft.
leaves no reasonable alternative to treating time as spacelike” (Quine 1960, 172).

Apart from arguably urging us to revise some of our most basic assumptions about space and time, special relativity has also been claimed to show that the tense-logical project must founder. Massey expresses the point succinctly when he writes that tense logic is “ill-advised because grounded in bad physics” (Massey 1969, 31), arguing that tense logic is bound to a pre-relativistic, Newtonian, empirically refuted conception of time. The central thrust of Massey’s challenge is that given SR, a notion of the present such as is required by tense logic cannot even be defined. Prior for one certainly took this challenge quite seriously, and some of his latest papers (e.g., Prior 1968 and 1970) deal with that issue, which is still the subject of an intense debate.¹ In this paper we wish to give a novel answer to the relativistic challenge by showing the feasibility of the tense-logical project with respect to both claims mentioned above. To this end, it will be important to separate clearly the two aspects of the relativistic challenge referring to these two claims: one should distinguish (1) the question, belonging to the philosophy of language, of whether a tense-logical language can work in the context of special relativity, from (2) the metaphysical question of whether presentism, claiming a metaphysically distinguished status of the present as that which alone is real, stands any chance in the face of the empirical success of special relativity.

These two questions have often been identified. Separating them will allow us to answer the relativistic challenge in two steps. In what follows, we will first show how a tense-logical language can work in special relativity, building upon previous work of our own (Müller 2002, 2004). We will then tackle the more difficult, metaphysical aspect of the relativistic challenge by showing how an indeterministic conception of ontological (causal) determination based on Belnap’s theory of branching space-times (Belnap 1992) offers a fruitful interpretation of presentism in the context of special relativity.

¹ Recent works on the problem of the present in SR include Mellor (1998), Müller (2002), and Rakić (1997).
2. Expressive adequacy of tense logic

Both the linguistic and the metaphysical aspect of the relativistic challenge can be illustrated by Figure 1, which gives a space-time diagram of two observers, \( A \) and \( B \), observing a distant flash. The solid vertical line indicates the \( t \)-axis of the diagram, which coincides with \( A \)'s world line. The world line of \( B \), who is moving relative to \( A \), is indicated by the dashed vertical line, which is at an angle to \( A \)'s world line. Observers \( A \) and \( B \) coincide (meet) at the diagram’s origin.

Special relativity gives a clear verdict as to which events are simultaneous for any observer at any given point on her world line. With respect to the origin, the simultaneity hypersurface of observer \( A \) coincides with the solid horizontal line, i.e., the diagram’s \( x \)-axis. For \( B \), the simultaneity hypersurface at the origin is represented by the dashed horizontal line, which is at an angle to that of \( A \). The linguistic challenge now comes about as follows: The distant flash, \( e \), is present for \( A \), while it is future for \( B \). Thus \( A \) can say truly, “The light is flashing now”, while \( B \) can say truly, “The light isn’t flashing now, but it will be flashing”.

Figure 1: Two observers, \( A \) and \( B \), and a distant flash, \( e \).
A and B thus say contradictory things—and still, both are right. This is an odd situation, and it is not rendered any more acceptable by a standard tense-logical formulation, which would look like this (taking $\phi$ to stand for the present-tense “The light is flashing now” and using the future operator F for “it will be the case that”):

**Observer A:** $\phi$

**Observer B:** $\neg \phi \land F\phi$

These two sentences together yield the contradiction $\phi \land \neg \phi$. Surely, a logical calculus that allows one to deduce a contradiction from true descriptions of a perfectly reasonable situation must be ill-conceived? This verdict is strengthened when one observes that the depicted situation, giving rise to the contradiction, is inherently relativistic: In Newtonian space-time, the simultaneity hypersurfaces for A and B at the origin necessarily coincide, blocking the contradiction.

This looks like a vindication of Massey’s complaint that tense logic is “ill-advised because grounded in bad physics”. However, there is a simple and, moreover, tense-logically natural answer to the purported difficulty. Tense logic takes the perspectival nature of assertions seriously and thus provides the natural resources to cope with the problem of the depicted situation.

In tense logic, like in any branch of modal logic, sentences are evaluated locally, with respect to a so-called index of evaluation, and that index can be shifted via modal operators in accord with the standard Kripke semantics for modal languages. For example, in linear tense logic, the index consists of a point in time, and sentences are evaluated in such a way that the present refers to the moment of evaluation. A tense operator like “it will be the case that” has the effect of shifting the index of evaluation (in that case, to the future). In this way, tenses and other temporally indexical expressions are handled easily.

The question of how the index of evaluation in a modal language looks like must be answered by considering the relevant indexicals. E.g., a language that allows spatial reference via the expressions “to my left” or “to my right” (such as English) needs to represent the spatial orientation of the speaker as part of the index of evaluation. Systematic alterations of that index allow us to resolve indexical references made by others. Thus, if you say, facing me, “There is a stone 1 m to my right”, I know that you are referring to a place that is 1 m to the left from where you stand. (Note that this
implies chaining two indexical references together: One reference to your position, the other to the left, relative to that position.) In order to make tense logic work in the context of special relativity, we need to specify the index of evaluation appropriately. As it turns out, all that is needed is to include the speaker’s frame of reference in the index of evaluation, and to introduce modal operators that shift that index.

To be fair, one has to concede that English does not have any corresponding indexicals—we do not distinguish grammatically between your frame of reference and mine. However, that is explained very easily: In our everyday situations (including the physicists’ laboratory!), there is no need for such a distinction. It simply does not happen that people pass each other with speeds anywhere near a significant fraction of the speed of light. When one looks at philosophical discussions that try to depict such situations, one invariable encounters scenarios like people riding their bikes at a speed of 0.9 \( c \). These rather inadequate examples just show—to sound a Wittgensteinian note—that relativistic encounters are not part of our life form. Certainly a life form has a history, and it can change. We can (can we?) imagine a human life form that lives in outer space and that has to deal with situations in which people do pass each other at really high speeds frequently. We can rest assured that that life form will have developed their language correspondingly. In the rest of this section, we will sketch one possible approach to such relativistic talk.

The basic idea of a relativistic tense logical language, or “logic of points of view”, is that (1) the index of evaluation includes an inertial frame and (2) there are available modal operators to shift that index. We will present

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2 For simple physical reasons, one cannot ride a bike or anything else at any significant fraction of the speed of light on the surface of the earth, since one would invariably start to escape the earth’s gravitational field. Frictional effects would cause anybody approaching very high speeds in the earth’s atmosphere to burn to ashes anyway.

3 Technology certainly can drive changes in the way we talk, and perhaps even in our conceptual scheme. E.g., medical advances may already have altered our conception of illness. To make an empirically unfounded guess: today, it seems that it is possible to classify a person as terminally ill even though that person does not show any symptoms of illness, while such classification was not possible a few hundred years ago.

4 That approach was inspired by some remarks in Prior’s (1968) paper on “Tense logic and the logic of earlier and later”, where he alludes to a “logic of points of view”. Some of Prior’s ideas have been developed in Müller (2002, written in German; cf. also the English article Müller 2004).
the simplest framework here, i.e., a propositional modal language, where
the propositions refer to what is true or false at space-time points. The basic
formal fact behind the logic of points of view is that the transformations be-
tween the inertial frames of special relativity form a group, viz., the (proper,
orthochronic)\(^5\) Poincaré group \(\mathbb{P}\). An element of that group is described by
10 real parameters: 4 for a spatio-temporal translation, 3 for rotation, and
3 for a Lorentz boost. A frame of reference can be described by an element
of the Poincaré group, interpreted as a transformation relative to some ar-
bitrary, fixed frame. The group structure secures that there is a transforma-
tion between any two frames, each such transformation has an inverse, and
composition of transformations is associative.

Formally, we take an index of evaluation to be a frame \(f\), and we associate
with each Lorentz transformation \(l \in \mathbb{P}\) from the (proper, orthochronic)
Poincaré group a modal operator \(\mathcal{L}\). For the identity element of the group,
\(I\), the corresponding modal operator, \(\mathcal{I}\), will result in no change of the index
of evaluation. The semantic clause for the modal operators builds upon the
notion of a model \(\mathcal{M}\), specifying for each atomic proposition and for each
space-time point whether the proposition is true there. The clause for the
atomic propositions is, accordingly:

\[(\text{atomic}) \quad \mathcal{M}, f \vdash \phi \text{ if and only if at the origin of } f, \phi \text{ is true according to } \mathcal{M}.
\]

The clauses for the propositional connectives are standard. The clause for
the modal operators reads as follows:

\[(\text{modal}) \quad \mathcal{M}, f \vdash \mathcal{L}\phi \text{ if and only if } \mathcal{M}, f' \vdash \phi, \text{ where } f' \text{ is the frame of reference } f \text{ transformed by } l.
\]

The group structure of \(\mathbb{P}\) then secures the following formal facts about the
modal operators:

- For each \(\mathcal{L}\) there is some \(\mathcal{L}^{-1}\) such that both \(\mathcal{L}\mathcal{L}^{-1}\) and \(\mathcal{L}^{-1}\mathcal{L}\) act as \(\mathcal{I}\).
  I.e., each modal operator can be “cancelled” by another modal operator.
  This generalises the fact that in tense logic, e.g., “it will be the case a day
  hence that it has been the case a day before” acts like “at present”.

\(^5\) We thus disregard spatial or temporal mirror images.
• Any two modal operators can be combined to form a single operator: $\Box \Diamond$ acts like $\Box' \Diamond$, where “o” denotes group multiplication in $\mathcal{P}$.

• The combination of modal operators is associative.

So far, the language sketched does not address the problem of the present at all. However, it is only a small step towards meeting the relativistic challenge. Among the modal operators, there is a (4-parameter) subgroup describing pure spatio-temporal translations by a space-time vector $\Delta \vec{s}$. Thus, the language has a full set of metric spatio-temporal determinations. Relative to any inertial frame, these determinations allow for a unique decomposition into a spatial and a temporal component—one can read the spatio-temporal vector $\Delta \vec{s}$ as a combination $(\Delta x, \Delta t)$. The language thus has the resources to interpret “presently” as “presently here or somewhere else”, i.e., as a purely spatial translation.

What about the mentioned problem of $A$ and $B$ saying contradictory things? A very general fact about communication with indexicals shows that (a) there is no inconsistency and even (b) how $A$ can use $B$’s statement in a meaningful way.

(a) By assumption, $A$’s and $B$’s frames of reference, $f_A$ and $f_B$, are different. In terms of the semantics sketched above, the fact that both $A$ and $B$ are right means that there is a model $\mathcal{M}$ such that $\mathcal{M}, f_A \models \phi$ and $\mathcal{M}, f_B \models \neg \phi$, which is absolutely unproblematic. (A basic tense-logical analogue would be obtained by considering, e.g., “It is raining now” and “It isn’t raining now”, uttered at different times.) The relativistic challenge has dissolved.

(b) Furthermore, our framework allows us to understand how $B$ can make good sense of what $A$ says. Quite generally, in indexical communication we have to presuppose that the hearer “knows where (at which index) the speaker is” (think of the example about left and right given above). In our case, this means that $B$ knows the transformation $l_{BA}$ from her frame of reference to $A$’s frame. If $A$ then says “at a spatial distance $\Delta x$, $\phi$” ($\Box \phi$), $B$ can prefix this sentence by the transformation $l_{BA}$, pulling it back to her frame. The resulting indexical sentence, $l_{BA} \Box \phi$, evaluates like a sentence $\Box' \phi$, where $\Box'$ is no longer a purely spatial translation. Thus in the example, $B$ can infer that $A$’s sentence, “the light is flashing now” means for her, “the light will be flashing”.

The language described above has many shortcomings, to be sure. It does not address the difficult question of how to interpret persisting objects in special relativity, nor have causal operators been introduced. For our purpose,
however, the important issue is just that the language is feasible, and that it meets the relativistic challenge easily. There is no problem, really, in talking about the present in a relativistic setting.

3. Metaphysical adequacy of tense logic:
Presentism in the context of special relativity

Before going on to discuss the metaphysical aspect of the relativistic challenge, we would like to express a doubt about the relevance of the whole discussion. In taking special relativity to raise a metaphysical question, that theory is afforded a metaphysical status, which presupposes a realistic attitude towards special relativity. Whatever one thinks about scientific realism, a minimal condition that any realistically interpreted theory must meet is empirical adequacy. Special relativity, however, is not empirically adequate in an unqualified sense. Of course, the theory is empirically highly successful within its domain of application—but that domain does not encompass all there is. Metaphysics, however, is about everything. Gravitation, the force that keeps our feet on the ground, lies outside the scope of special relativity, and the same holds true of many other phenomena. Thus, the metaphysical impact of special relativity can at best be limited, and the following discussion should be read in that light. If the solution offered here seems artificial, one should not forget that the problem is an artificial one to start with.

Given that proviso, we wholeheartedly agree that the language part met the easier of the two challenges put forward against the tense-logical project. You talk about your present, I talk about mine, and we can still reach agreement. That is nice, but none too spectacular. What about ontology? Can the relativising strategy of the previous section be adapted to meet the metaphysical challenge? The prospects don’t look good—the notion of a relativised ontology will probably appear incoherent. Are there other options?

In what follows, we will first try to phrase the metaphysical challenge in a way that allows for a formal treatment. We will then sketch the known options of solving that formal task. Finally and most importantly, we will suggest a novel approach that we claim successfully answers the metaphysical challenge.

3.1 From metaphysical intuition to a formal question

It has been said above that tense logic is often connected with the metaphysical view of presentism, according to which that which is real at any
moment, is what is present at that moment. The task thus is to show a way in which presentism may be sustained against the background of special relativity. The basic problem here is to say what “the present” is at any moment. Taking “moment” to be “space-time point” (the only reasonable option), the problem depicted in Figure 1 becomes pressing: relative to the origin, there is no such thing as “the present”—there is A’s present, B’s present, and so on. In the last section, these relativised notions of the present were enough to secure communication between A and B—but can a frame-relativised notion of “the real” be appropriate? Hardly anybody thinks so. Gödel for one strongly expressed his view that a relativised ontology is a contradiction in terms: “existence by its nature is something absolute” (Gödel 1949, 258n). Rather than opting for a maverick position, we will accept that aspect of the metaphysical challenge: Whatever the question is, it can’t have a frame-relative answer. That is: If A’s and B’s positions coincide,⁶ then A and B have to give the same answer when asked what is real, i.e., ontologically present with them.

Formally, we are thus after a one-place notion of “the present”, given a space-time point, or, equivalently, a two-place relation of “being ontologically present with” between space-time points, which is frame-independent. Furthermore, it will be uncontroversial to claim that (i) each event is ontologically present with itself (reflexivity) and (ii) if e is present with f, then f is also present with e (symmetry). Many philosophers have argued that the relevant notion must be transitive, too: If e is present with f, and f is present with g, then e is present with g as well.⁷ We will accept this intuition. Thus, the sought-for notion of presentness must be an equivalence relation that

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⁶ Another one of these tellingly inadequate ways of talking. Coinciding with the position of another observer at a high relative speed means annihilation for both, of course. This is more than just a witticism, since it points to the fact that the notion of “spatiotemporal coincidence” operative in special relativity refers to an extended region of space-time, and it may be a pragmatic matter how big that region is. Einstein conceded that point when even in his original definition of simultaneity (Einstein 1905) he referred to events “in the immediate neighbourhood” (“unmittelbare Umgebung”) of the observer.—While our own solution will be to argue for a notion of the present as a spatio-temporally extended region, we will not exploit the pragmatic aspects here mentioned. By this we only strengthen our position, allowing less resources for meeting the challenge. Thereby we do not wish to claim that the pragmatic resources available are useless.

⁷ Cf. e.g., van Benthem (1983), Stein (1991), Clifton and Hogarth (1995), and Rakić (1997).
is not based on frame-relative notions. What are the candidates? How can one define the present in special relativity?

3.2 Available options

At first sight, the prospects look none too good. An argument made rigorous by van Benthem (1983, 25f.; cf. also Stein 1991 and Clifton and Hogarth 1995) shows what the trouble is: Any relation definable on the basis of special relativity alone must be invariant under all of the theory's automorphisms, including the full Poincaré group and contractions. Now these automorphisms can be exploited systematically to show that, given an equivalence relation $R$ and just two points $x \neq y$ such that $xRy$, we already get $x'R'y'$ for any $x'$ and $y'$. Thus, van Benthem's theorem states that the only equivalence relations definable on the basis of special relativity are trivial, viz., the identity relation and the universal relation. Taking “present with” to mean identity would lead to the verdict that each event is present only with itself, while according to the universal relation, any two events would be present with one another. The first option thus leads into straightforward “solipsism of the present moment”, while the second leads to a notion of “the present” devoid of any discriminatory significance.

Special relativity alone, e.g., in the form of Robb's (1914) axiomatisation, does not provide the resources for a satisfactory definition of “the present”, i.e., of the sought-for relation of “ontologically present with”. Contrapositively, this means that any satisfactory definition of the present on the basis of special relativity will have to avail itself of an extension of that theory. Which extensions are reasonable? In the literature, Rakić's dissertation (Rakić 1997) is the most advanced study of possible extensions so far.

Rakić first makes the requirements on a satisfactory definition of “the present” more stringent. Basically, she is after a notion of the present as a space-like hypersurface, which may seem reasonable enough. She then points out that the mere fact that such a hypersurface cannot be defined does not preclude adding such a hypersurface. One is thus led to the question, not whether “the present” can be defined, but whether adding “the present” to the basic structure of special relativity has any unwelcome ef-

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8 Cf. Mundy (1986) for a perspicuous presentation of an equivalent, much simpler system.

9 Our own solution will deviate from Rakić's already at this point.
fects. It turns out not to: Rakić proves that a satisfactory notion of “the present” may be added conservatively to Robb’s axiomatisation of the SR causal relation.10

So far so good. One may be satisfied with this result for a number of reasons. E.g., one may welcome the technical result because one believes that there is a physical explanation of a preferred frame after all, even though one outside special relativity. General relativity in many models provides the necessary resources in terms of a definable notion of “cosmic time”. Given a cosmic time for any event, two events can be defined to be present with one another if and only if they happen at the same cosmic time. “The present” is then just the set of all point events that happen at the same cosmic time as the origin. This is a perfectly good equivalence relation with a clear, satisfactory interpretation.

Whatever the ultimate justification for preferred hypersurfaces, and no matter how nice the technical result mentioned above is—one needs to concede that by going that way, one is leaving orthodox special relativity. We are sympathetic to those who would hold that this may be a good thing, but if we are looking for a sustainable notion of presentism within special relativity, we need to search for other options.

3.3 A novel approach based on branching space-times

What are candidates for a less “intrusive” addition to the framework of SR? Add we must, but we may be able to add something less controversial than preferred hypersurfaces. There seem to be two good options: Adding indeterminism, and adding persisting objects. We will not explore the option of introducing objects, since that would presuppose a lengthy discussion of persistence in SR.11 Thus, the option we wish to explore here is whether indeterminism can offer the additional resources needed for a satisfactory definition of the present in special relativity.

Indeterminism is the thesis that there is more than just one possible future. That thesis needs to be spelled out carefully in the context of SR, as SR itself is one of the very few deterministic theories of physics (cf. Earman (1986), esp. Chap. 4, for discussion). There is, however, a well worked-

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10 This means that all new theorems of the extended theory include the new presentness relation, i.e., the set of theorems that can be formulated in the old theory is left unchanged. Cf. Rakić (1997, 50ff.) for formal details.

11 Cf. Friebe (2005) for a promising approach, which starts by clarifying the concepts of perdurantism and endurantism with respect to SR.
out formal theory of indeterminism in the context of special relativity, viz.,
the theory of *branching space-times* developed by Nuel Belnap (1992). That
theory generalises the well-known theory of branching time by allowing for
histories (complete courses of events) to be not just linearly ordered sets, but
space-times with a relativistic causal ordering.12 Branching space-times will
form the background for what follows. Rather than giving a full presenta-
tion of the formalism (for which cf., e.g., Belnap (1992, 2002, 2005)), we
will only highlight informally those aspects of the theory that are immedi-
ately relevant in the present context.13

In employing an indeterministic theory for giving a formulation of pre-
zentism compatible with SR, we first have to specify what presentism means
in an indeterministic universe. In view of our aim of establishing presentism,
we are looking for a *single* relation between space-time points that allows for
two interpretations, viz., as “present with” and as “real relative to”. We will
argue that the sought-for relation is the relation of being “ontologically co-
determined with”, and we will specify ontological determination in terms
of indeterministic causation: Two events are ontologically co-determined if
and only if they have the same (indeterministic) causes.

Our interpretation of indeterministic causation is indebted to Belnap’s
new theory of causation (Belnap 2005), which uses the formal framework
of branching space-times to determine the *causae causantes*, or originating
causes, of indeterministic events.14 Belnap’s theory singles out so-called *basic
transitions*, consisting of a point event and one of its immediate possible fu-

12 The branching time framework was developed by Prior in the 1950ies and 1960ies
in his attempt of giving a semantics for the future-tense operator of tense logic. For
a good overview of the so-called Prior-Thomason semantics, cf. Belnap et al. (2001),
Chap. 6–8.
13 Rakić in her dissertation (1997) already provided a first attempt at employing
branching space-times for a clarification of the problem of the present in the face
of special relativity. However, her approach does not use the full strength of the
theory. Rather, she first gives an interpretation of “the present” in terms of preferred
hypersurfaces, as outlined in the previous section, and then extends that reading to
the branching framework. Our approach will proceed differently, using the branch-
ing aspect directly.
14 As Belnap argues convincingly, the relata of the causal relation are in general
not just events, but transitions, consisting of an initial *I* (“first this”) and an out-
come *O* (“and then that”); an event may be viewed as a transition with an empty
initial. For our purposes, it will be sufficient to consider events as that which is
caused; this corresponds to the first stage of Belnap’s analysis. Cf. Belnap (2005) for
details.
tures, as the basic building blocks of the causal structure of our indeterministic world. For any event $e$, the set of its *causae causantes*, $CC(e)$, is defined to be the set of those indeterministic basic transitions in the past of $e$ that were responsible for bringing about $e$ instead of one of its alternatives.\(^{15}\)

We will follow Belnap in accepting a claim that may be rather controversial: Deterministic events (those occurring in every history) are not caused in any non-trivial sense—they happen anyway. Only indeterministic events have non-trivial causes, i.e., a non-empty set of *causae causantes*. For a deterministic event $e$, $CC(e) = \emptyset$, so in some sense, “nothing happens”. Following this line of thought, we can make good sense of a notion of objective change that is also tied to indeterministic events: There is change from event $e$ to event $f$ if and only if, given that $e$ and $f$ can occur within a single possible course of events (i.e., in some history), $e$ and $f$ do not occur in exactly the same histories, i.e., $CC(e) \neq CC(f)$. Change conceived of in this way is not a language-relative thing, but something rooted in the objective, indeterministic structure of our world. We do not need to trouble ourselves with trying to find out the most basic predicates with which to describe the world in order to capture change (a gruesome task, as every good man knows)—we just need to ask whether something else could have happened. If yes, we have change; if no, we don’t.

It is now just a small step to arrive at our indeterministic conception of presentism—we just need to accept a variant of the thesis that “time involves change”, so that the present is that during which there is no change. The present of $e$ can then be determined as that region of space-time in which there is no objective change relative to $e$. Formally, we define:

$$e \text{ PRES } f \text{ if and only if } CC(e) = CC(f).$$

The present conceived of in this way, as the region that is ontologically co-determined with, and thus real in relation to, the origin, by having the same *causae causantes*, can have various geometrical shapes. Figure 2 illustrates two possibilities:

\(^{15}\) For the formal definition of $CC$ as well as for illustrations, cf. Belnap (2005).
Figure 2: A: The present of \( e \) is equal to the shaded, extended region, including the solid lines, but excluding the dashed lines. Dots mark indeterministic events. B: The present of \( e \) as a space-like hypersurface.

If the indeterministic events acting as initials of basic transitions form a discrete set, which appears like a natural assumption, then the present will be a region of space-time that has both a space-like and a time-like extension (cf. Figure 2.A, which includes three indeterministic events). In the extreme case in which indeterministic events are distributed densely, we can recover the preferred hyperplanes mentioned above (cf. Figure 2.B).\(^{16}\) At first, this consequence of our definition may appear weird: How could something be present which is in the causal future? Again, separating the two aspects of the relativistic challenge pays off. The linguistic, frame-relative notion of the present indeed needs to single out a space-like hypersurface. Ontologically, however, if nothing happens, time is just a coordinate. An ontological notion of time by assumption only comes into play once there is objective change.

With respect to the situation of Figure 1, the question of whether the distant flash is ontologically present with the origin can now be answered. The answer depends on the distribution of indeterministic events in the causal

\(^{16}\) The situation is actually more complicated than that. If the densely distributed indeterministic events act independently, the situation can collapse to what above was called “solipsism of the present moment”: the present of an event can consist of just that one event. A preferred hyperplane is obtained if the indeterministic events act in a correlated fashion, exhibiting what Belnap has called “EPR-like funny business”. Cf. Belnap (1992) for a discussion of “branching along a hyperplane”, and his (2002, 2003) for a discussion of the notion of “funny business”.
past of the flash and of the origin. The two are present with one another if $CC(\text{flash}) = CC(\text{origin})$, but not otherwise—and this holds for both observers, $A$ and $B$.

Apart from now fully answering the relativistic challenge, we claim that our approach also allows us to understand what is good about the “block universe” view of Minkowski space-time, according to which everything is there at once, so that the only available reading of “the present” is the whole “block” of Minkowski space-time itself.

This intuition does not only mesh well with the formal result reported in the previous section, according to which the universal relation is, apart from solipsism, the only definable candidate notion for “present with”—the intuition is also reproduced in our theory! If our world is completely deterministic, so that a branching model contains only one branch, i.e., a single Minkowski space-time, then all events have the same set of $\textit{causae causantes}$, viz., the empty set. Thus, no change, no time, and everything is ontologically present with everything else. Only indeterminism can save us from the block universe view.

To summarise our results:

- In special relativity, with or without indeterminism, a tense-logical language is not only formally unproblematic, but also pragmatically useful, as speaker and hearer can resolve indexical references in communication.
- Ontologically, bare Minkowski space-time may be viewed as a “block universe” with the trivial notion of presentness as the universal relation. However, given indeterminism, a non-trivial and positively illuminating notion of the present can be defined.
- Thus, far from foundering on the special theory of relativity, the tense logical project should be seen as a fruitful challenge to that theory. Tense logical considerations urge us to extend the theory of relativity in a way that allows us to recapture both the scientific results on which that theory is based, and our deeply held commonsense notions of space and time, central to our own conception of ourselves as real agents. Indeterminism provides the crucial ingredient that solves the problem of defining the present in special relativity.
References


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